WORKSHEET 7
DEFLECTED SHAPE & BENDING MOMENTS

An introduction to the relationship between the bending moment and the deflected shape of a structure using the Push Me Pull Me models on Expedition Workshed.

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INTRODUCTION

Engineer's Bending Theory relates the kinematics (curvature) with the stresses developing along a beam of length L.

In this tutorial we will deal with point loads and how to relate the curvature of the structure to the bending moment diagram, making decisions such as which is the tension side (i.e. side which is stretched) and how curvature changes along the beam.

DEFLECTED SHAPE & BENDING MOMENT

When a structural element is subjected to bending, it curves causing one side to stretch and the other to contract. This extension and contraction mobilise internal stresses (tensile and compressive, respectively) across the depth of the structural element, which when summed are referred to as the bending moment.

Since curvature is directly proportional to bending moment, where the element remains straight, there are no bending stresses and thus no bending moment.

As long as the material remains elastic, when the load is removed, the bending stresses reduce to zero.

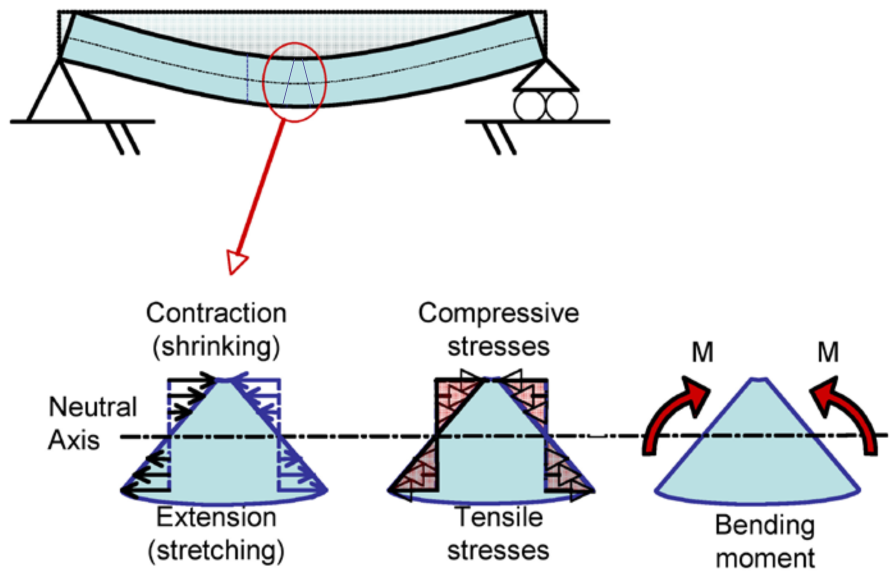


Fig. 1. A schematic relating curvature to bending stresses and eventual bending moment.

DEFLECTED SHAPE & BENDING MOMENT

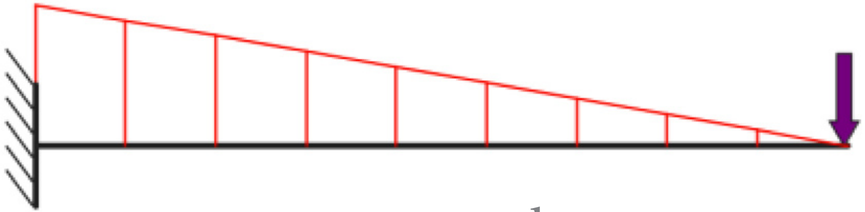
If the flexural rigidity of the beam (EI) is constant, then the curvature is directly proportional to the bending moment as a result of the applied loading.

To the first order, the curvature function can be integrated twice to give the deflected shape along the beam, by applying suitable boundary conditions.

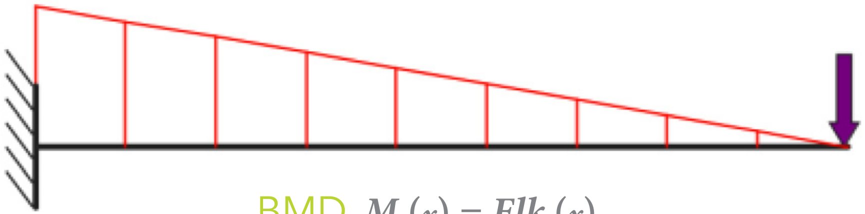
Obviously these are standard results but simple rules of thumb can help you sketch approximate bending moment distributions from deflected shapes, just as you have used applied loads to sketch deflected shapes.

$$\frac{M}{I} = Ek \rightarrow M(x) = EIk(x)$$

A laterally applied point load causes a linear curvature distribution. Curvature functions and bending moment diagrams are plotted on the tension side (i.e the side that stretches).



Curvature $k(x) = \frac{1}{EI} M(x)$



BMD $M(x) = EIk(x)$

Fig. 1. A deflected cantilever along with the corresponding bending moment and curvature diagrams accompanied by the associated equations.

DEFLECTED SHAPE & BENDING MOMENT

Therefore since curvature is directly proportional to the bending moment diagram, from the kinematic boundary conditions discussed in the previous tutorial we can make some deductions for the statics.

a) At fixed supports, where rotation is restricted, the beam curves and a bending moment exists as the resultant. For equilibrium a reaction moment also exists, equal and opposite to the bending moment at the support.

b) Pins, whether at supports or between beams, allow free rotation and therefore the curvature and bending moment on the elements on either side of the pin are zero.

c) The curvature along a beam changes sense (i.e. from sagging to hogging and vice versa) at the point of contraflexure. The bending moment there is zero. That means for the specific loading and configuration the presence of a pin in modelling would not have made a difference at this location.

d) Bending moment 'flows' around rigid joints, in order to satisfy rotational equilibrium

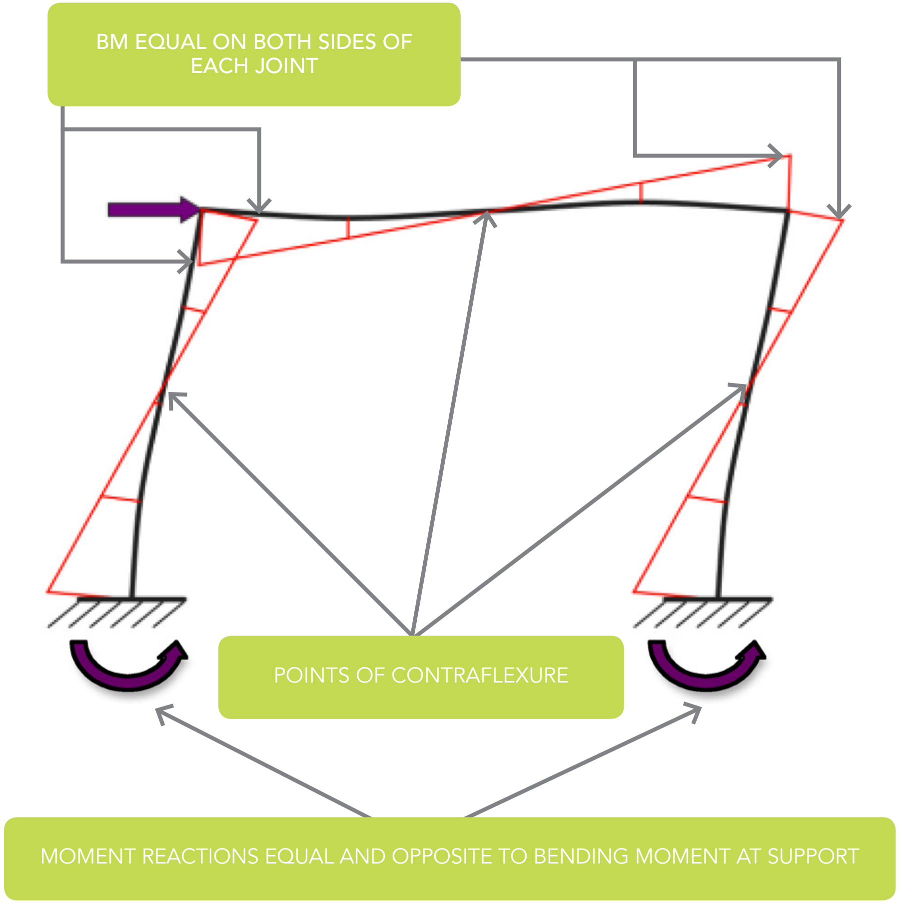


Fig. 2. A fixed portal frame deflecting under a horizontal load applied to the top left corner. The bending moment along the beam has then be sketched in red with annotations to explain trends.

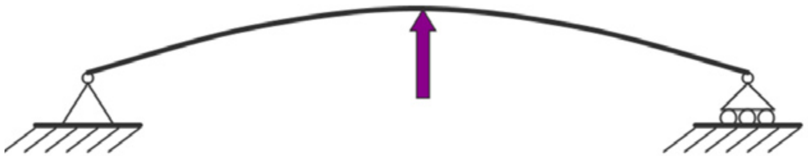
QUESTION TIME

1. **Sketch the approximate bending moment diagrams (on the tension side), given teh point load and deflected shape of the beams below.**

   Use the Push Me Pull Me models to verify your answers.

a.

b.



c.

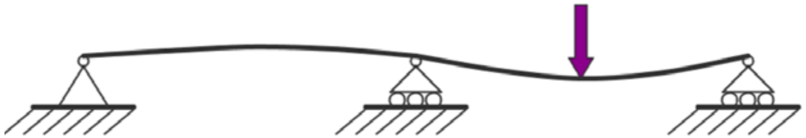
d.

e.

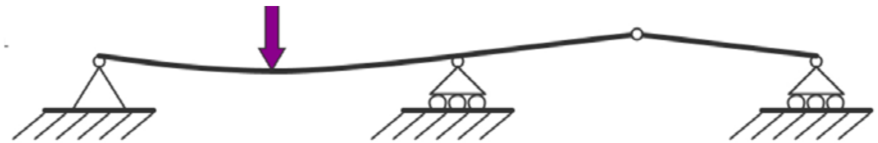
f.

QUESTION TIME

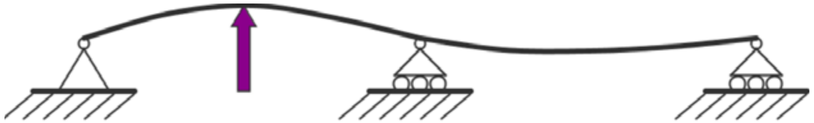
g.



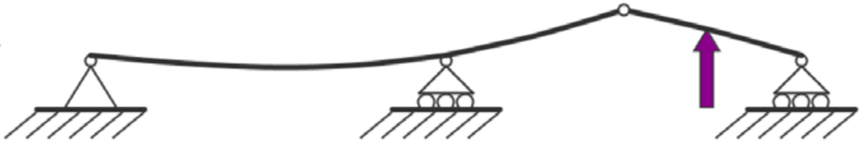
j.



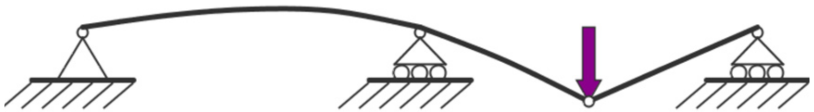
h.



k.



i.

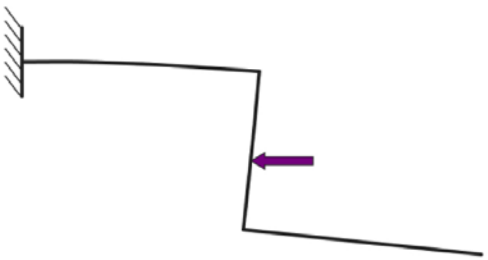


QUESTIONS

2. **Continue on, but this time for frames.**

Keep using the Push Me Pull Me models to check your answers.

a.



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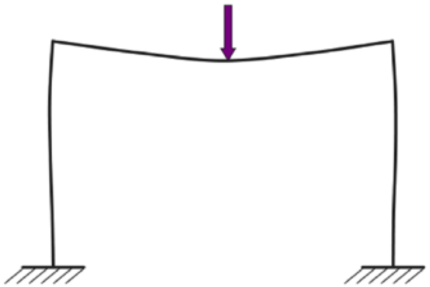
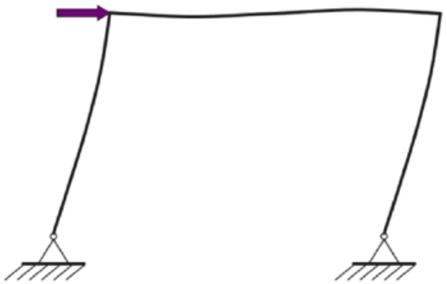


b.



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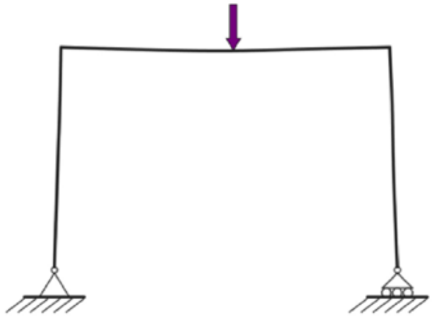

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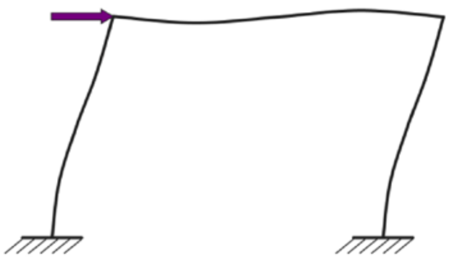
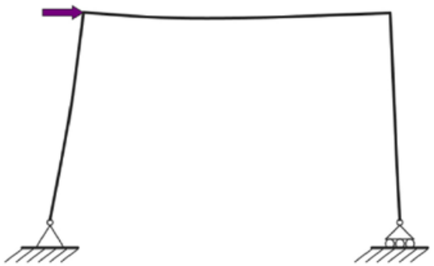
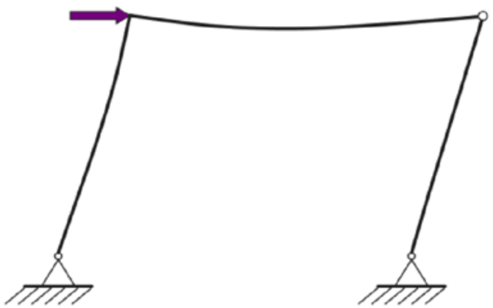


f.



QUESTIONS

g.



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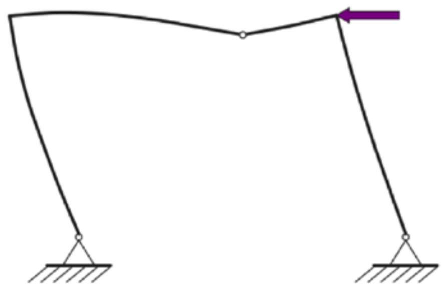

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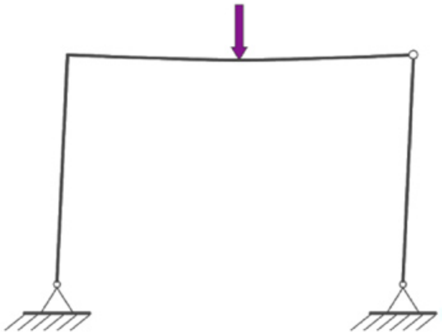

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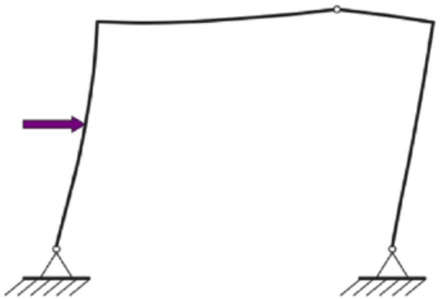
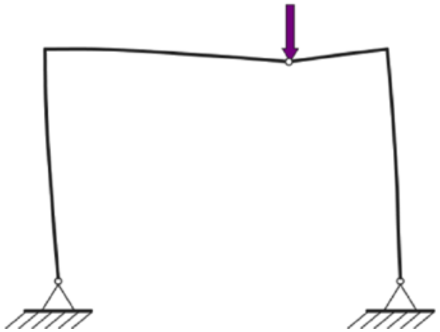
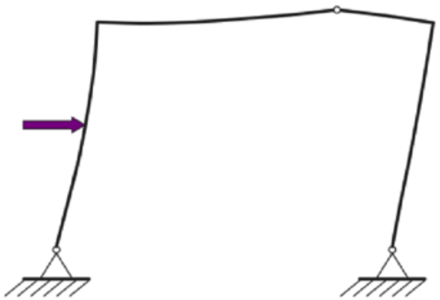


i.

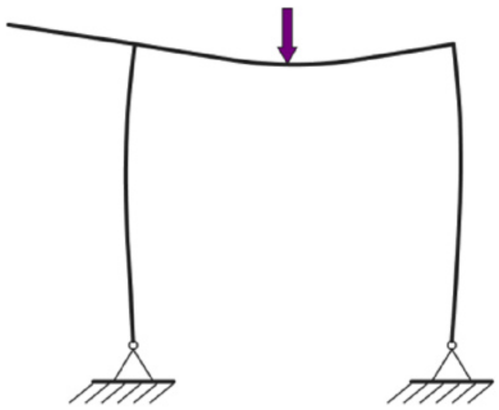


l.

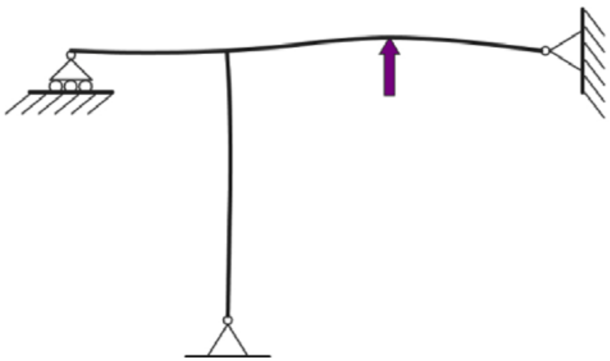


QUESTIONS

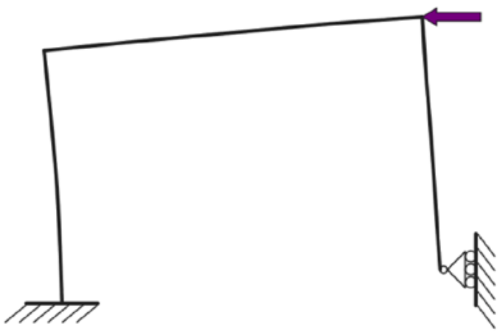
m.



o.



n.



p.