WORKSHEET 6
BENDING MOMENTS & SHEAR FORCE

An introduction to the relationship between bending moments and shear forces using the Push Me Pull Me models on Expedition Workshed

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INTRODUCTION

The shear stresses in a cross-section arise in order to balance lateral support reactions and provide equilibrium in that direction. Since these shear stresses are parallel but not in line with the reaction force, a couple is formed and equilibrium is maintained by the presence of a bending moment counteracting the couple.

In the previous tutorials we were able to deduce the approximate shape of bending moment diagrams (BMD) from the deflected shape of the structure and the shear force diagram (SFD) from the direction and position of support reactions.

One can also deduce the shape of the SFD from the respective BMD as the size and direction of the shear force along the beam is the first derivative of the bending moment with respect to its length (i.e. the slope of the bending moment diagram).

BMDs & SFDs

Taking for example the cantilever in Fig. 1 under the applied load we can easily see that as it deflects downwards it curves, stretching at the top.

If we want to have a closer look into the stress resultants, we need to split the structure into two free bodies as seen in Fig. 2.

Fig. 1. Deformed shape and load and reactions.

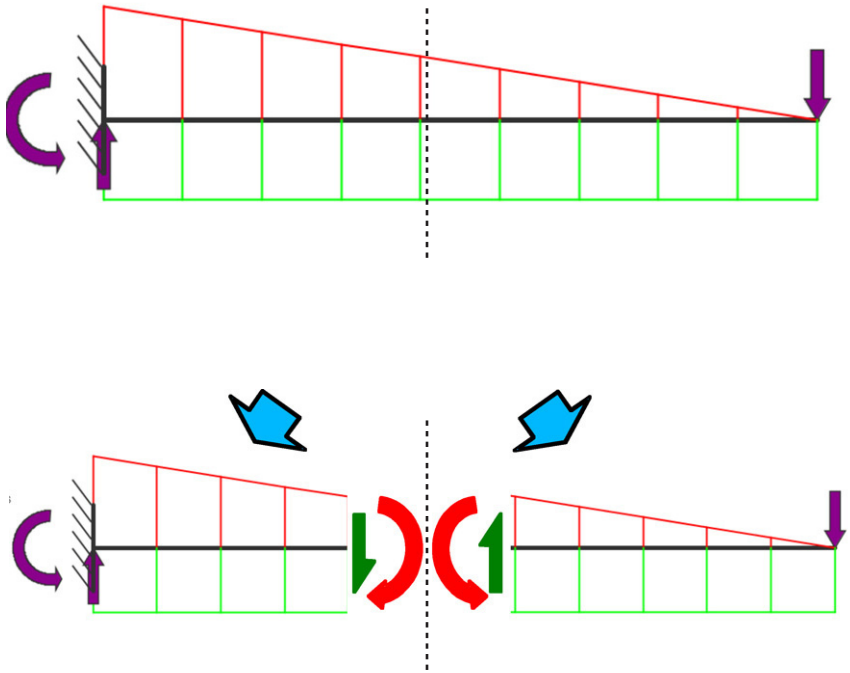


Fig. 2. Top: Beam Statics BMD (red) and SFD (green). Bottom: BM sense and SF direction at a cut .

BMDs & SFDs

With reference to the lower diagram in Fig 2, at either side of the cut, a shear force and a bending moment arise which are equal in magnitude and opposite in direction (or sense) from their counterparts in the other free body. Looking at the left-hand free body the shear force at the cut has to balance the vertical reaction force at the support. The bending moment at the cut has to balance the reaction moment, which itself is reduced by the shear force at the cut. On the right-hand free body the shear force at the cut has to balance the applied load. The bending moment at the cut has to balance the bending moment which is the resultant of the applied load and the shear force at the cut.

What is clear is that in this case the shear force remains constant irrespective of where the cut (to separate the two free bodies) is made, while the bending moment reduces linearly as the cut is made further from the support. The reduction in the bending moment is due to the increasing effect of the shear force on the reaction force couple which balances the reaction moment.

Therefore that the shear force distribution is related to the rate of change of the bending moment distribution along the beam.

$$V(x) = -\frac{dM(x)}{dx}$$

Fig. 3. The variable x describes unit length along the structural member and the negative sign is present because of the standard convention we have been using in these tutorials:
BMD: Positive when there is tension below (or inside for frames)
SFD: Positive when shear force is clockwise.

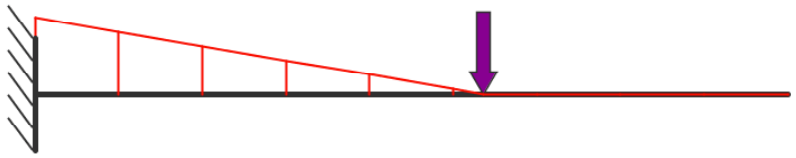
Quiz: If a linear bending moment distribution is associated with a constant shear force distribution, what is the bending distribution associated with a linear shear force diagram?

Quiz: Could there be a situation where there is only bending and no shear in a structural element?

QUESTION TIME

Given the bending moment diagrams, sketch the associated shear force diagrams using the usual convention and then use Push Me Pull Me to verify your results.

a.



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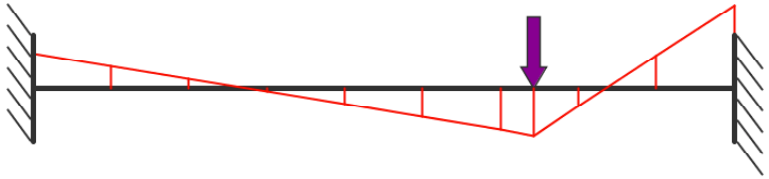

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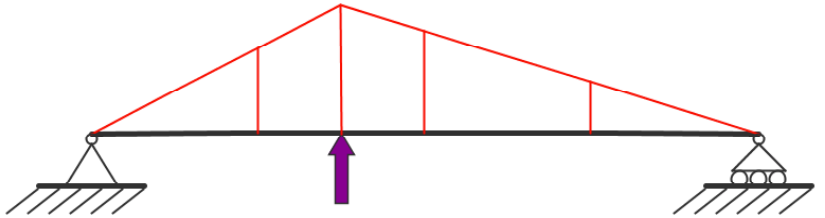

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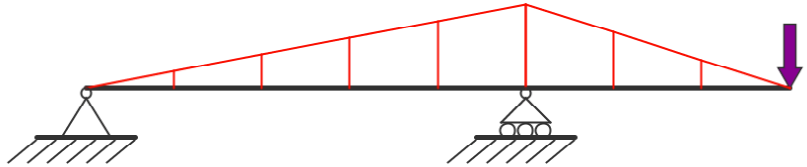

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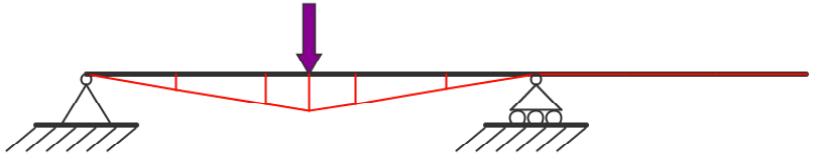

QUESTION TIME

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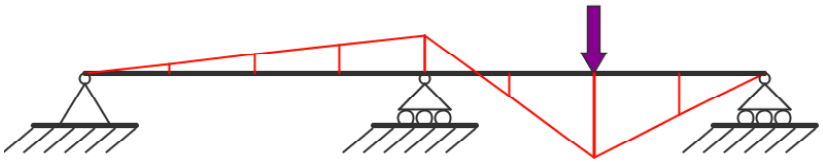

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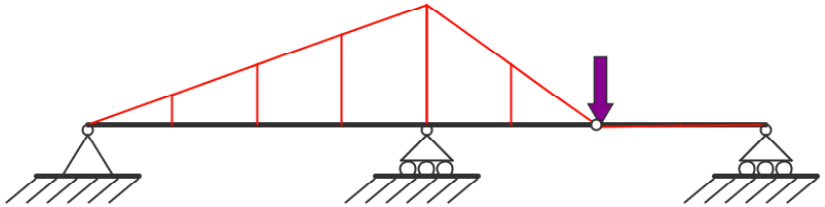

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