WORKSHEET 2
DEFLECTIONS OF BEAMS

An introduction to understanding the deflection of beams using the Push Me Pull Me models on Expedition Workshed

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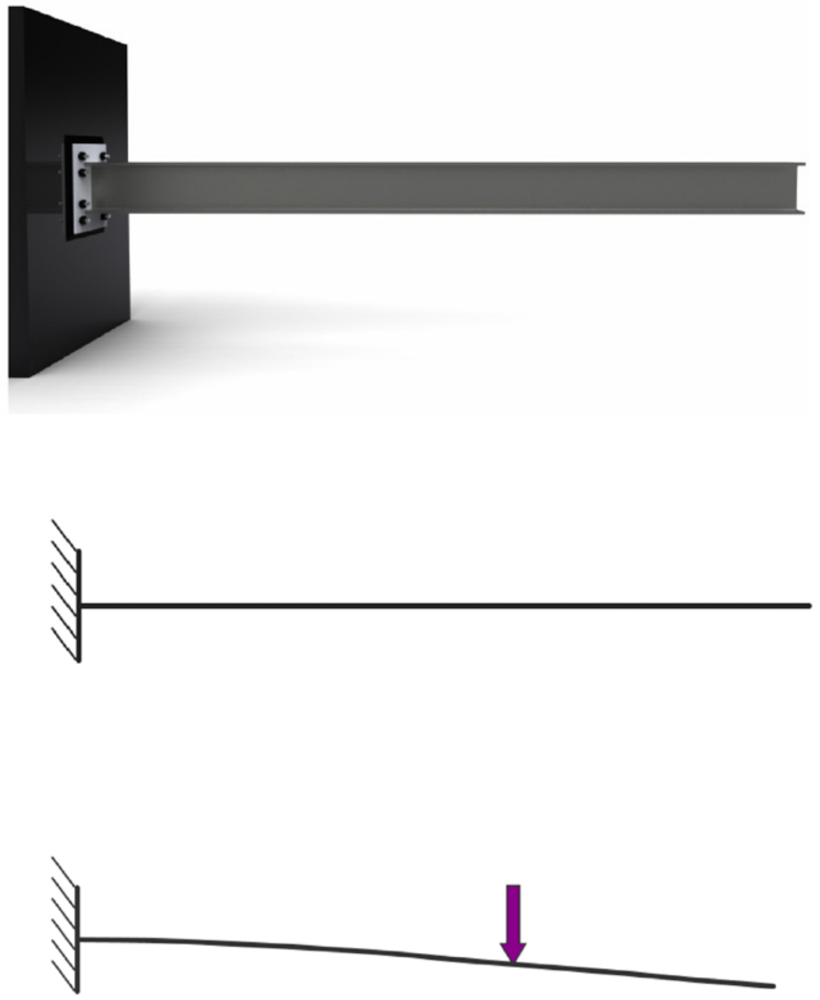
PRODUCED WITH FUNDING FROM THE ROYAL ACADEMY OF ENGINEERING'S
NATIONAL HE STEM PROGRAMME.

INTRODUCTION

Understanding how structures deform and deflect upon mechanical loading is a key part of understanding structural behaviour. The deflected shape of a structure can provide information on the bending stresses, since the way the curvature varies along the beam is related to the bending moment.

Using Push Me Pull Me models, this tutorial will help students understand how an action will cause a given structure to deflect for a given set of support conditions.

The first part of this tutorial provides worked examples of different loaded structural configurations, with hints and tips on how to estimate the deflected shape. Then, students are asked to sketch the approximate deflected shape of different structures and use Push Me Pull Me models to verify their answers.



CANTILEVER

A cantilever is the simplest single-support, isostatic structure. Natural cantilevers are seen in the form of trees everywhere. Find the cantilever on the Push Me Pull Me Expedition Workshed website:

WORKSHED > MODELS > PMPM > CANTILEVER BEAM

Looking at the render of the real structure (Fig.1), it has a single support on the left-hand side. This support is modelled as a fixed support in Fig. 2.

Kinematically, this means that the left-hand end cannot move horizontally or vertically. Furthermore the rotation of the left hand end is suppressed. This is a modelling assumption that can be considered valid, due to the fact that the support connection has bolts located at some distance from the beam's neutral axis, preventing translational and rotational displacements.

Therefore under the application of any load, anywhere along the beam, the left hand end will remain at its unloaded position.

Fig 1. Cantilever Push Me Pull Me model

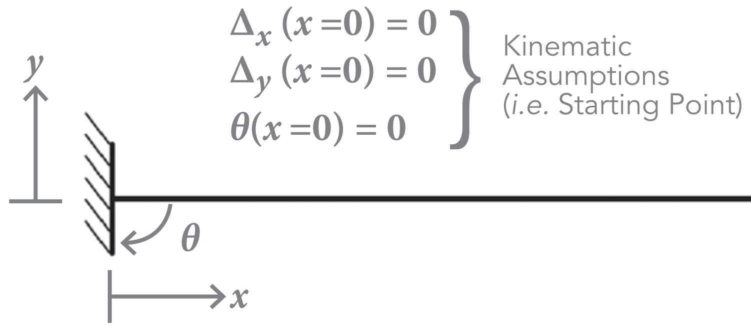


Fig 2. Line model of a cantilever showing the boundary conditions for horizontal and vertical deflection, and for rotation at the left hand end of the structure.

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CANTILEVER

Assuming the structure is stable out-of-plane, applying a lateral (vertical) load anywhere along the beam will cause it to bend and deflect in the direction of the load.

Observations:

a) The left hand side (x=0) has not moved or rotated.

b) The largest vertical deflection occurs at the right hand tip of the beam (x=*L*).

c) The beam bends (flexes) between the fixed end and the point of load application. Beyond that point the beam remains straight.

Quiz: At which point along the structure would you place the load to maximize the vertical displacement at midspan?

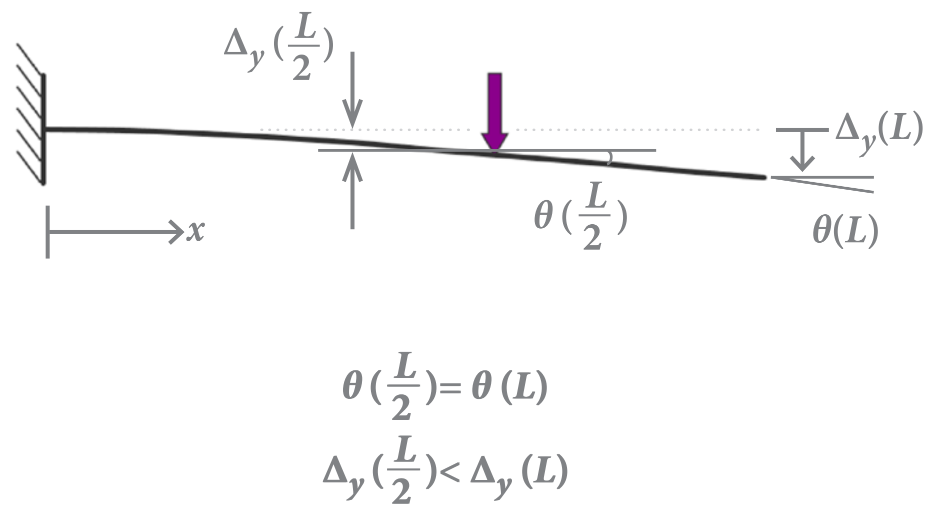

Fig 3. Diagram showing the deflection of a cantilever when a point load is applied halfway along its length.

SIMPLY SUPPORTED BEAM EXAMPLE

A simply supported beam is another common type of isostatic structure this time being supported at both ends.

As depicted in Fig.4, the left-hand side is bolted very close to the neutral axis. The support on the right-hand side is very similar, the only difference being the slotted cleat angles connecting the beam to the wall allow (some) horizontal displacement of the beam.

On the Expedition Workshed website go to:

WORKSHED > MODELS > PMPM > SIMPLY SUPPORTED BEAM

Looking at the kinematics of the beam and assuming out-of plane stability, the beam can be modelled with a triangular simple support on the left that suppresses translations (i.e. horizontal and vertical movement) and on the right with a roller support that suppresses only vertical displacements (Fig.5). Rotations are free at both ends and upon the application of a load anywhere along the beam, the ends will rotate.

Fig 4. Simply Supported Beam Push Me Pull Me model of Expedition Workshed.

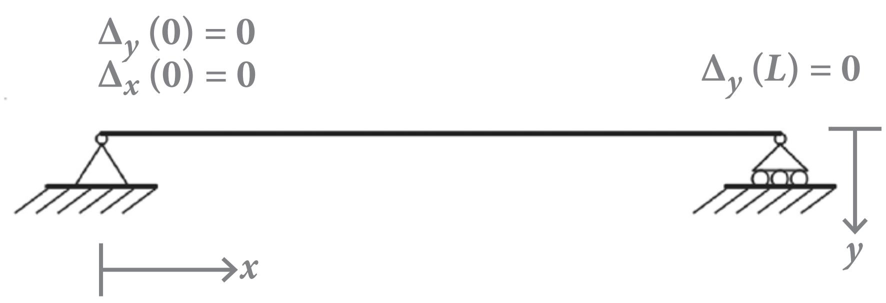

Fig 5. Line model of a simply supported beam showing the boundary conditions at both ends.

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SIMPLY SUPPORTED BEAM

Upon the application of a lateral load the beam will deflect in the direction of the load as a result of bending.

Observations:

a) Applying the vertical point load anywhere along the beam does not change the position of the two ends. They simply rotate to accommodate beam flexure (Fig. 6).

b) If the vertical load is applied at midspan, the maximum diplacement and flexure will coincide with the point of application (Fig. 6).

c) If the point load is applied anywhere along the beam, except midspan, then the maximum vertical displacement does not coincide with the maximum curvature, which occurs at the point of load application (Fig. 7).

d) As there is nothing to prevent the ends of the beam from rotating, the curvature at either end is always zero.

Quiz: Under the application of a moderate lateral load, why doesn't the right hand support move in?

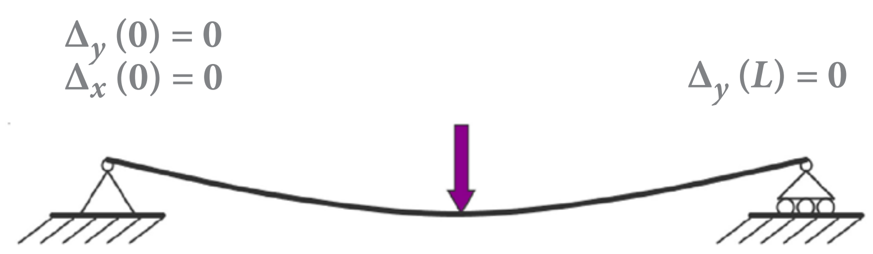

Fig 6. Line model of a simply supported beam with a point load applied at the centre of the span.

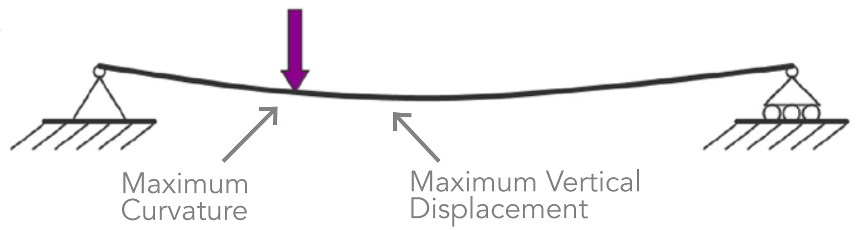

Fig 7. The deflection of a simply supported beam under the action of a non-central point load.

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SIMPLY SUPP. BEAM WITH CANTILEVER END

An extension to the simply supported beam is the simply supported beam with a cantilever at one end. In real life this structural configuration can be described by a simple cleat angle connection on one end and a support column between the main span and the cantilever as this is depicted in Fig. 8.

This second support provides vertical restraint but allows the beam to rotate. It should be noted however that the beam on the left-hand side and the right-hand side of the support rotate simultaneously since the beam is continuous over the support.

On the Expedition Workshed website this can be found at:

WORKSHED > MODELS > PMPM >
SIMPLY SUPPORTED WITH CANTILEVER

By clicking on draw model the mathematical model is revealed (Fig. 9), with a simple support on the left (vertical & horizontal restraint) and a roller support on the right (vertical restraint only).

Fig 8. Simply Supported Beam with Cantilever Beam End Push Me Pull Me model.

Fig 9. Line model of a simply supported beam with a cantilever end. Note that the pin on the roller support is under the beam. If the pin were in the beam, the right-hand end would simply collapse.

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SIMPLY SUPP. BEAM WITH CANTILEVER END

Upon the application of a point load in the main span (ie. between the two supports), the beam would behave like the simply supported example, assuming the self-weight of the beam is not considered, with the beam bending between the two supports which allow free rotation, and the cantilever deflecting but remaining straight. The downward curvature of the main span of the beam is referred to as `sagging' (see fig. 10).

When the point load is applied anywhere along the cantilever (ie. to the right hand side of the right-hand support), the whole beam is seen to be deflecting and curving upwards, which is referred to as 'hogging'.

Quiz: In the case where the point load is applied on the cantilever, is the portion of the beam to the right of the right-hand support straight?

Trivia: The term 'hogging' comes from the back of a pig (aka 'hog') which resembles the deflected shape of a beam that is curving upwards.

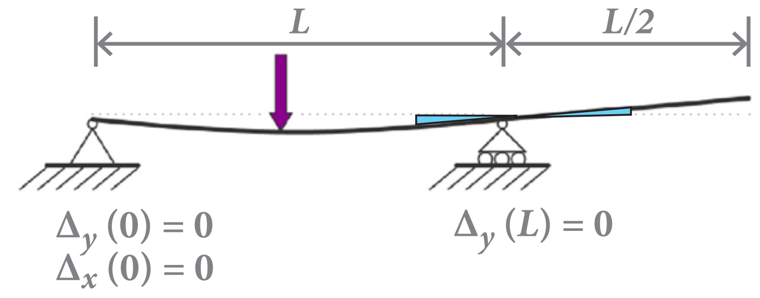

Fig 10. Simply Supported Beam with Cantilever line model featuring a load in the main span, demonstrating again the boundary condition and also the notation of the case.

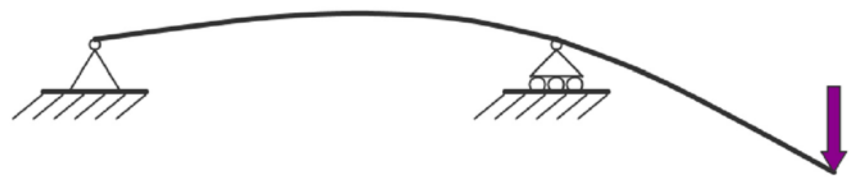

Fig 11. The same line model, this time with the load shifted to the end of the cantilever. Note the change from sagging in Fig 10. to hogging in Fig 11.

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ENCASTRE (FULLY FIXED) BEAM

An encastré (also known as a 'fully fixed') beam is one in which rotations and displacements are suppressed at both ends. Practically, this can be achieved by fixing the steel beam depicted in Fig. 12 with a wide plate which is bolted on the wall. The bolts not only provide vertical and horizontal restraint but also by placing them some distance away from the beam's neutral axis, they prohibit rotation at the two ends.

Fig 12. Encastré Beam Push Me Pull Me model

Quiz: The encastré beam is said to be hyperstatic. What does that mean and to what degree is this the case?

The beam can be found on the Expedition Workshed website at:

WORKSHED > MODELS > PMPM > ENCASTRE BEAM

The mathematical model of the 2D beam restricts all displacements at the ends, making the response of the beam stiffer.

Due to the presence of rotational restraints, there are two points of contraflexure on either side of the applied load, as can be seen in Fig.13.

At a point of contraflexure, curvature changes sign as seen in Fig. 13.

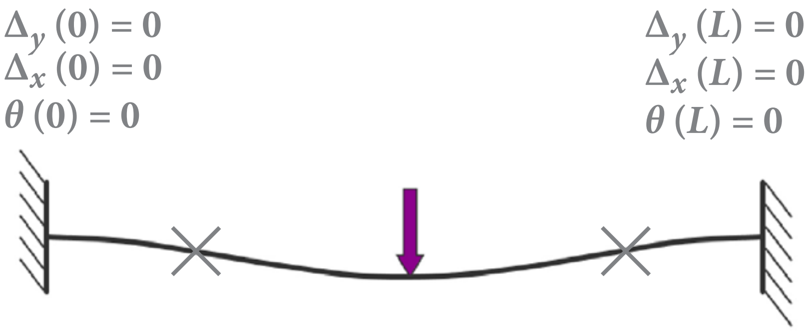

Fig 13. Encastré beam line model with a load in the centre of the beam. The 'X' marks correspond to the points of contraflexure.

Quiz: if the point load is applied at midspan, where are the points of contraflexure located?

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CONTINUOUS BEAM WITH INTERNAL PIN

Sometimes continuous beams are designed such that a long beam is connected via a plate joining the webs of the girders as shown for the right span in Fig. 14. The presence of the small connecting plates on either side of the web, allow the free rotation at the connection, since the connection acts like a door hinge.

The beam can be found on the Expedition Workshed website at

WORKSHED > MODELS > PMPM > TWO SPAN SIMPLY SUPPORTED BEAM CONTINUOUS OVER CENTRAL SUPPORT WITH INTERNAL PIN

The mathematical model of the 2D beam suppresses the vertical and horizontal displacements on the left support and the vertical displacement at the central and right support. The pin in the middle of the right span models the web plate connection assuming free rotation at that location.

Quiz: By applying the load at several different locations along the beam, do you see any points of contraflexure?

Fig 14. Continuous Beam with Internal Pin.

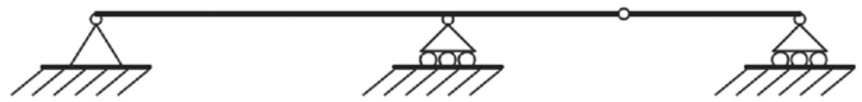

Fig 15. A line diagram of the same beam.

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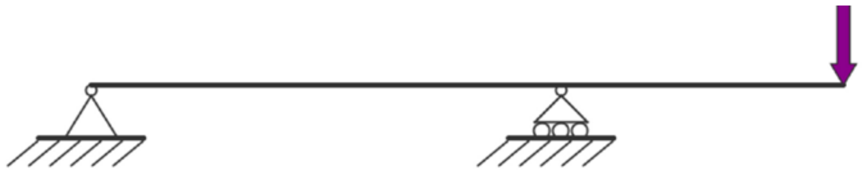
QUESTIONS

Being able to sketch approximate beam deflection shapes is an extremely important skill for a civil engineer.

Try the examples below and then use Push Me Pull Me models to verify your results.

a.

d.



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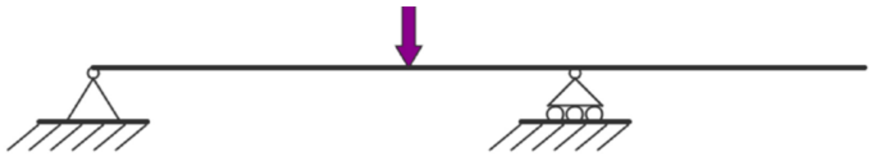
e.



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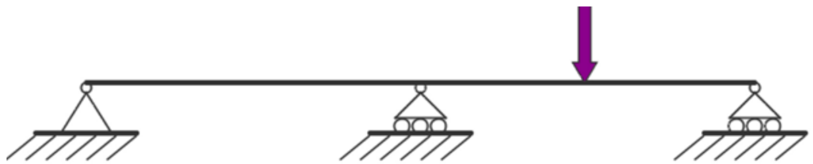

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QUESTIONS

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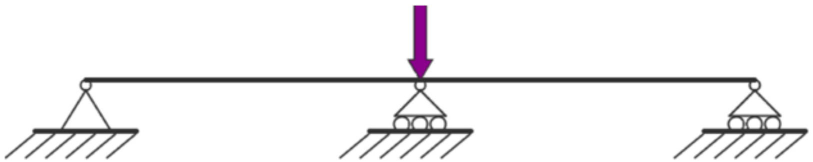

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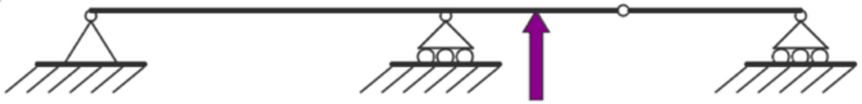

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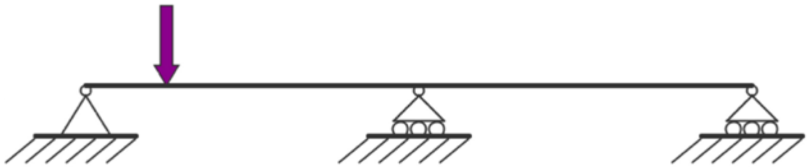

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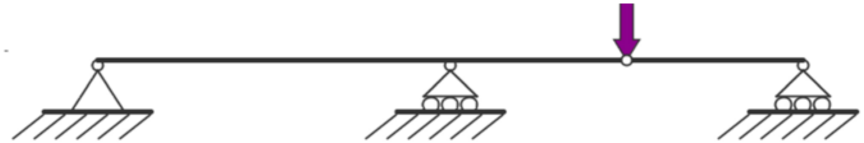

i.